Prospects for Destructive Self-Induced Interactions in a Vortex Pair

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The vortex wakes of large transport aircraft can pose a hazard to smaller following aircraft in the vicinity of airports during landing and takeoff operations if certain separation guidelines are not observed. In order to reduce the hazard potential, and thereby the separation distances, efforts are being made to find more rapid wake-dissipation mechanisms. In this paper, numerical simulations are made of the three-dimensional time-dependent instabilities that might be initiated in a vortex pair by sinusoidal displacements of the filaments. The objective of the study is to find those displacements and phase angles that produce the most rapid destruction of the vortices. It is concluded that, of the wave patterns tried on the one pair of wake filaments, the only instability mode that leads to destructive interactions of the vortices is the Scorer-Crow process.

Nomenclature

a	= wave amplitude
A	$= a/b$
b	= span between vortices
ds	= initial length of vortex segments
r_c	= radius of vortex core
t	= time
T	= dimensionless time $= tU_\infty/b$
$T\Gamma$	= time parameter $= (tU_\infty/b)(\gamma/bU_\infty)$
U_∞	= velocity of aircraft
x	= flight direction
y	= spanwise direction
z	= vertical direction
X,Y,Z	$= x/b, y/b, z/b$
β	= roll amplitude of wake-generating aircraft
γ	= vortex strength
Γ	$= \gamma/bU_\infty$
ϕ	= phase angle between waves on port and starboard vortices
θ	= rotation angle of wave plane
ρ	= air density

Subscripts

c	= vortex core
i	= initial condition

Introduction

THE persistence of the lift-generated wakes trailed by large aircraft can pose a hazard to smaller aircraft that inadvertently encounter the high-energy parts of the wake. Currently, the technique used to maintain safe operations is to avoid the wakes. Avoidance of the more energetic wakes during landing and takeoff operations requires certain separation distances between aircraft to allow time for the wake vortices to move out of the flight corridors, decay, or both so safety is not compromised. In this way, the hazard due to wake vortices is reduced to a negligible level but the separation guidelines have a significant impact on the capacity of airport runways to accommodate traffic. Therefore, efforts[1-6] have been under way for some time to find ways by which the velocities in the lift-generated wakes can be reduced to a safe level at smaller separation distances. As an extension of previous research, the present study investigates the rates of wave growth and the destructive capability of the instabilities that are initiated on a vortex pair by sinusoidal displacements. That is, the prospects for initiating instabilities that produce large distortions of the wake-vortex filaments are investigated to find those which decompose lift-generated vortex filaments into chaotic shapes and motions. The resulting wakes then usually decay more rapidly than when the vortices are in straight lines and simply diffuse. One such instability for a single vortex pair was pointed out by Scorer.[7] He observed that the single pair of wake vortices trailed by jet aircraft flying at high altitudes (where water in the exhaust condenses to mark the vortices) take on nearly periodic waves. The amplitude of the waves grows while the planes in which the waves are located rotate until they are inclined at about 40 deg to the vertical. At about the time that the wave planes have reached such a position, the amplitude of the waves has increased to the extent that, because of the rotation of the wave planes, the high-speed cores of the vortices approach each other closely. If the instability continues to completion, the cores then disconnect and link up across the pair to form irregularly shaped loops of vortex filaments which accelerate the dispersion and decay of the wake. This process was first attributed to buoyancy differences.[7] It was later shown by Crow[8] that the wave growth is brought about by velocities induced by the vortices on themselves by their curvature and proximity to one another. Crow's analysis also showed that small-amplitude waves would grow over a wide range of imposed wavelengths.

The foregoing instability can be triggered by any technique that places nearly periodic waves that are in phase on a pair of trailing vortices; e.g., by an aircraft in pitching or plunging motion. The case when the imposed waves are out of phase was also studied by Crow[8] (his antisymmetric case) and found to be unstable. The out-of-phase case could be initiated in flight by executing a series of roll oscillations with the wake-generating aircraft. Although the symmetric case that converts the two line vortices into a series of irregularly shaped loops is observed with great regularity at high altitudes, an example of the antisymmetric or out-of-phase instability appears to be quite rare. To the author's knowledge, the effect of roll oscillations on a vortex wake was first studied in a series of flight tests conducted by Barber and Tymczyszyn[5] with an L-1011 and a B-747 transport aircraft. Additional information and a discussion

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of the results are provided by Burnham[6] on the ground-based measurements which accompanied the flight tests. The flight tests covered a wide range of configurations in straight and level flight and in both roll and pitch oscillation. Most of the tests used the aircraft with some of the spoilers deployed and with the flaps and landing gear extended. The resulting wakes immediately behind the aircraft consisted therefore of multiple vortex pairs and large amounts of turbulence.

In the case of the B-747, the wake-hazard alleviation achieved (as measured by a following probe aircraft) with roll oscillations is attributed[9] to the interaction of the multiple vortex pairs. Since this paper is restricting its attention to the dynamics of a single pair, those experiments are not pertinent here.

In the case of the L-1011, wake alleviation was not enhanced by roll oscillations of the wake-generating aircraft. Subsequent theoretical analysis[9] and examination of the motion pictures taken of the smoke-flow visualization parts of the flight tests indicated the reasons for the different result. It appears that the fairly dominant wingtip vortices trailed by the L-1011 causes the complex wake which exists immediately behind the aircraft to become organized into a single vortex pair a short distance further behind the aircraft. When the L-1011 was put into a roll-oscillation maneuver of about a 7 deg amplitude, the flight experiment[5] and the numerical simulation[9] both showed that the wave amplitude grows while the wave planes rotate (through about 90 deg) until the wave planes of both vortices are horizontal. Such a vortex configuration appears to then be stable because no further rotation or wave growth occurs even though some minor vortex distortions do occur as time progresses. As a consequence, wake alleviation is not enhanced by roll oscillation of the L-1011.

Since the purpose of the theoretical study[9] was to provide an explanation of the flight results, further attention was not given there to the growth characteristics of the antisymmetric or roll-oscillation case. The cases studied did indicate that a different kind of wake dynamics is achieved when different roll amplitudes are input into the numerical simulation. The research described in this paper was therefore undertaken to determine which wave inputs lead to destructive interactions, and to be sure that a usable wake-alleviation scheme was not being overlooked. The investigation began by considering the details of only the out-of-phase case. Subsequent results suggested that the numerical simulations be broadened to include the entire range of phase angles between the waves on the port and starboard vortices. Such an investigation would indicate whether any initial wave patterns, besides the inphase or symmetric one, lead to destructive self-induced vortex interactions. If any such new patterns are found, their implementation as wake-alleviation schemes would then have to be addressed.

Numerical Method

The present numerical technique is the same incompressible inviscid technique used in Ref. 9, which is patterned after those used by others.[1,10,11] That is, the wake vortices are broken into straight line segments that approximate the curved filaments. The shape and locations of the vortex lines are monitored by following the points where adjacent segments or links are connected. The velocity components of these points are calculated as a summation of the separate contributions of all of the vortex links in the flowfield, except for the two that touch the point itself and thereby have singular contributions. Since the velocity induced by the two adjacent links is not included, the vortex lines are essentially cut off at a distance of one link length ds on both sides of the point in question. The length of the link and, consequently, the cutoff distance appears to have a small or secondary effect on the motion of the filaments. In order to minimize the magnitude of the approximations being made, the link length is chosen so that the prediction obtained with the present numerical method for the velocity of propagation of a vortex ring composed of straight-line links agrees with the closed-form expression[12] for a vortex ring with a small but finite core. The core is assumed to have uniform vorticity over a diameter d_c comparable with the vortices trailed by transport aircraft. Agreement is achieved with the present numerical method when $ds = 0.36d_c$. Based on these values, a link length of $ds = 0.2b$ is used throughout the present investigation. The symbol b is the initial spanwise separation between the vortices. The foregoing approximations are discussed in more detail in previous studies[1,8-11,13-16] where applications of the filament method have been made to a variety of different problem areas. Even though the present numerical method is not an exact analysis of vortex dynamics, the method has been found to produce both qualitative and quantitative results that represent actual laboratory and flight experiments. Furthermore, when the computations are repeated using a different link length, the results are the same within plotting accuracy if the curvature of the filaments is large compared with link length. Experience with these numerical methods indicates that the results are insensitive to link length and cutoff distance if the foregoing guidelines are followed.

The computations are begun by placing waves of amplitude a_i on the filaments in the vortex pair so that, at the beginning of each case, the waves on the port (or left) vortex filament have a phase angle ϕ between 0–180 deg relative to the starboard vortex. For comparison purposes, the initial amplitude a_i of the waves is set by the magnitude expected at the wingtips when the wake-generating aircraft rolls through an angle $\beta = 7$ deg. The value of 7 deg was chosen because that is the magnitude of the oscillations used to study wake alleviation in the flight tests.[5,6,9] The pitch oscillations used to generate the wave pattern for the inphase case are assumed to be of a magnitude such that the same displacement magnitude is achieved at the wingtips as when the aircraft undergoes roll oscillations. Hence, those cases are also given a β label to signal that the amplitude of the waves is comparable. Different initial amplitudes are then taken as multiples or fractions of the 7 deg case. All of the wavelengths are held fixed at six spans.

In order to reduce the computation time, only five wave cycles were considered in the analysis. The beginning and end of each wave set was blended smoothly into the straight-line portions of the vortices by use of one-quarter cycle of $\sin^2$ variation in the displacement of the vortex links. Flexibility of the wavy portion of the filaments was enhanced by placing two spans of links between the ends of the waves and the single link that projects off to the far reaches of the wake. In the figures to be presented, only the three center waves on each filament are shown because they are believed to be insulated well enough from the starting and stopping parts of the waves to represent the dynamics to be experienced by an infinite wave train. This conclusion is supported by the fact that a significant difference between the shape of the center and of the two adjacent waves on a filament is not detectable.

Since the magnitude of the circulation is the same for the entire length of both vortices in the pair, the circulation is combined with the time to form a time parameter. That is, the strength of the vortices in dimensionless form $\Gamma = \gamma / bU_\infty$ is combined with the dimensionless time $T = tU_\infty / b$ to form a time parameter $T\Gamma$. It is to be noted that time is the only variable in the time parameter and the other quantities are only scaling factors. The results presented in the figures to follow can then be readily applied to a vortex pair of any strength by simply dividing the displayed time parameter by the dimensionless circulation. For example, since a value of $\Gamma = 0.1$ approximates aircraft in their cruise configuration, division of the values presented for $T\Gamma$ by 0.1 yields the dimensionless time or spans behind the generating aircraft at which the results are applicable. Similarly, since $\Gamma = 0.2$ ap-

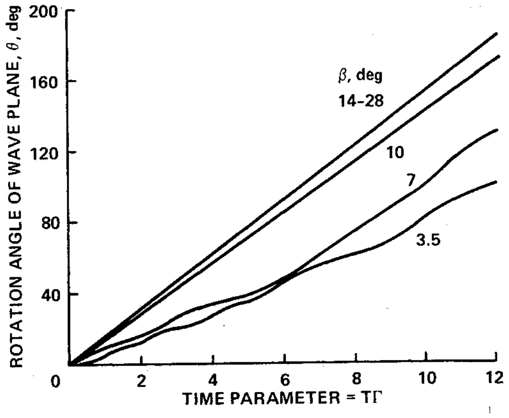


Fig. 1 Variation with time of rotation of plane of sinusoidal waves impressed on an isolated vortex with various initial amplitudes.

proximates transport aircraft in their landing configuration, division by 0.2 (or multiplication by 5) yields the time or location at which the results should be expected.

Dynamics Instigated by Various Wave Patterns

Isolated Vortex

Previous studies[13,14] of isolated vortex filaments have considered the motion of a vortex with various initial shapes and also have derived shapes that will move without change as time progresses. Since the published results have shown that a vortex filament that has sinusoidal waves along its length will move without much change in shape, the purpose of Fig. 1 is to illustrate how the rotation rate is affected by the amplitude of the waves. It should first be noted that the plane of the vortex waves rotates counter to the direction of the vortex rotation (i.e., retrograde direction) and that essentially no change in the amplitude of the waves occurs. As pointed out by Hama,[13] the sinusoidally shaped vortex rotates as a unit with small distortions taking place as time progresses. In the present analysis, it was observed that oscillations occur in the vicinity of the wave crests and troughs which consist of small changes in the shape of the waves in the direction normal to the wave plane. Since the angle of the wave plane was measured along the line from the crest to the trough, the rotation angles plotted in Fig. 1 reflect a change in shape together with any change in rotational speed. It is observed in Fig. 1 that when β is 14 deg or more, the rotation rate is approximately constant with time and initial amplitude even though the waves do curl a little at the crests in the direction perpendicular to the wave plane. At smaller initial wave amplitudes, both short- ($\beta = 3.5$ deg) and long-term ($\beta = 7$ deg) variations occur in the apparent rotational rate. An explanation was not found for the various idiosyncrasies in the rotation rates of the wave planes. These results provide some background characteristics on the rotation rate and on the changes in wave shapes from the initial sinusoidal one in order to help differentiate between the dynamics brought about by sinusoidal displacements on the filaments and by mutual interactions between the filaments.

Vortex Pair: Waves in Phase at $\phi = 0$ deg

The motion of a vortex pair when displaced in a sinusoidal manner has been studied by the use of several different numerical approximations.[1,8-11,15,16] As mentioned previously, the initial wave pattern on the filaments for this case could be thought of as having been generated by executing pitching or plunging motions with the wake-generating aircraft. Figure 2 presents the shape of the vortex filaments at three different times after the beginning of the event. Also, the time parameter $T\Gamma = (tU_\infty/b)(\gamma/bU_\infty)$ indicated with each figure represents a scaled distance of travel of the wake-

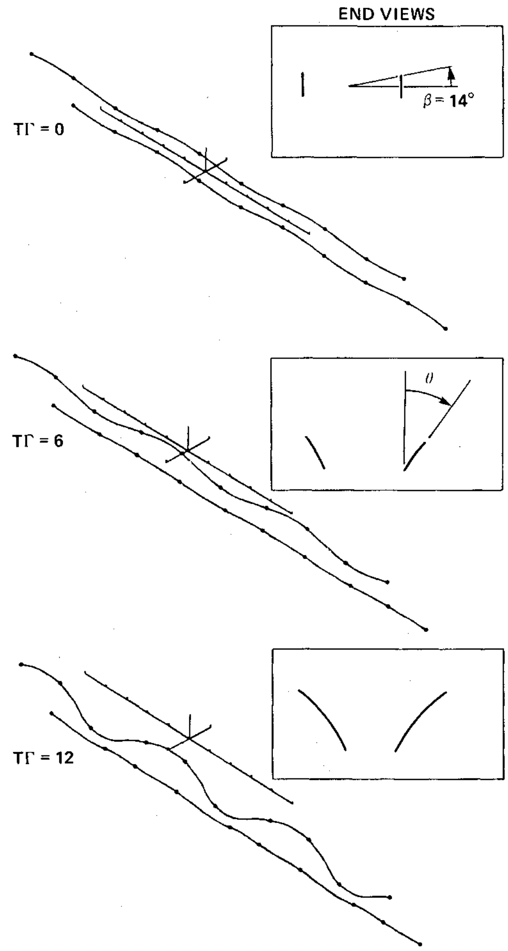
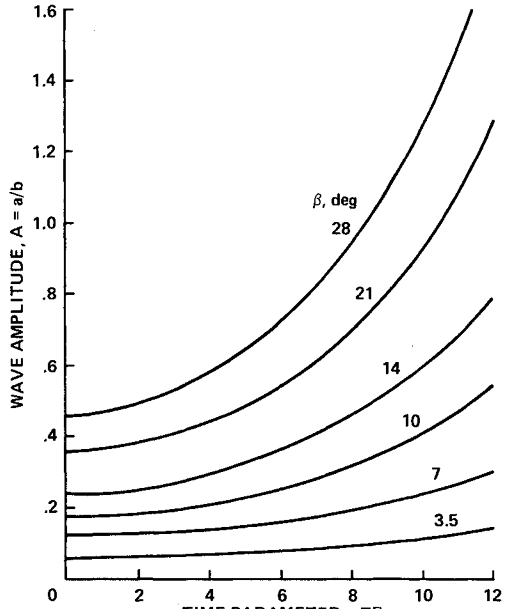


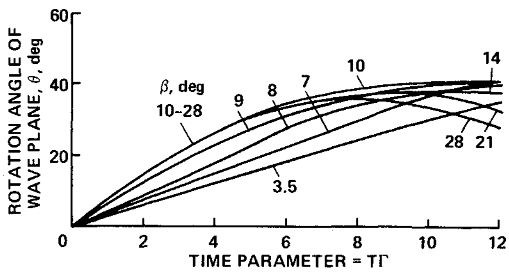
Fig. 2 Oblique and end views of vortex pair which has an initial displacement of sine waves that are in phase ($\phi = 0$ deg); wavelength = $6b$, initial amplitude = $0.23b$ ($\beta = 14$ deg).

generating aircraft from the beginning of the event. If the dimensionless circulation Γ is 1.0, the time parameter is the same as the vortex spans. The wakes are displayed as an end view and as a 30 deg/30 deg/90 deg oblique view to yield a three-dimensional perspective of the wake. The end views are plotted at four times the scale used for the oblique views to aid clarity. The present numerical method was used to calculate a range of input wave amplitudes to study the characteristics of the in-phase wave pattern. These results are summarized in Figs. 3 and 4 for the wave amplitude $A = a/b$ and the angle of the wave planes θ. The larger initial amplitudes lead to faster growth rates and to different rotation rates of the wave planes; see Figs. 3a and 3b. The decrease observed in the angle of the wave plane θ when the time parameter $T\Gamma$ exceeds about eight occurs because of the way that the angle is measured. That is, the measurement is made on a secant from the crest to the trough of the wave so that, as the wave grows rapidly in the trough region, the apparent plane of the wave becomes more vertical. It is also noted in Fig. 3b that at the smaller initial amplitudes, the wave planes rotate at different rates. However, if β is over 14 deg, the wave planes rotate at about the same rate independently of the amplitude.

If the dynamics of the interaction were linear, the curves in Fig. 3a would collapse to a single line when the ratio of the instantaneous amplitude to the input amplitude a/a_i is plotted as a function of the time parameter. As indicated in Fig. 4, the wave growth is nearly linear with the input amplitude during the initial stages of the event. As time progresses, however, the larger inputs appear to be more productive because they are the ones that bring the troughs of



a) Peak-to-trough amplitude.



b) Rotation angle of wave planes from initial vertical attitude.

Fig. 3 Wave characteristics as a function of time for various input amplitudes when waves are in phase ($\phi = 0$).

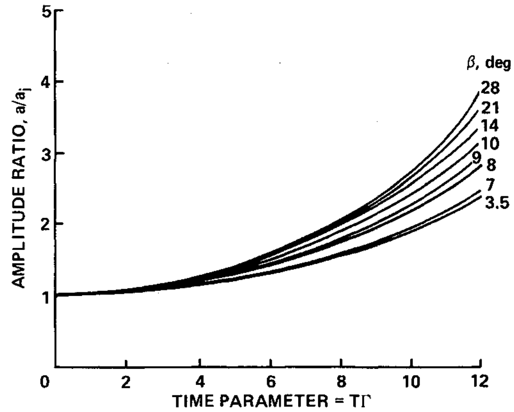


Fig. 4 Wave amplitude ratio a/a_i as a function of time to test for the linearity of the vortex dynamics to input amplitude when input waves are in phase ($\phi = 0$ deg).

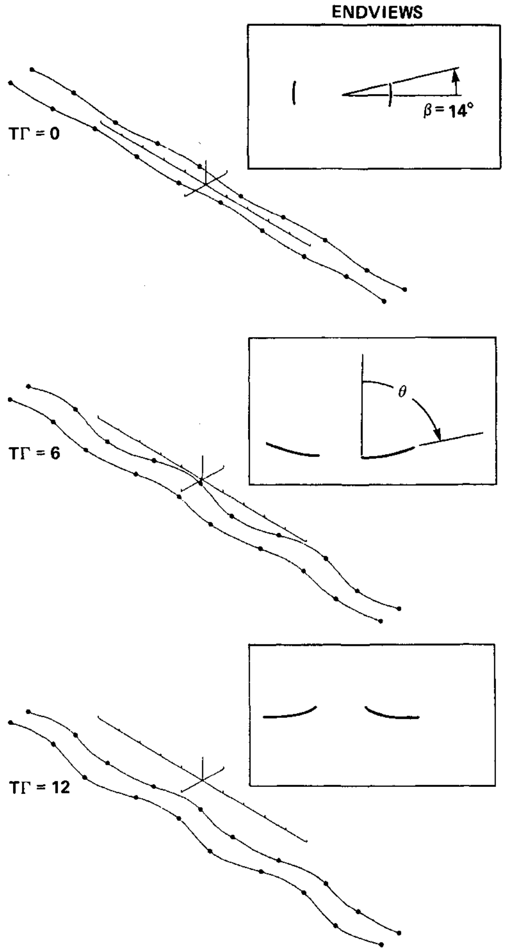


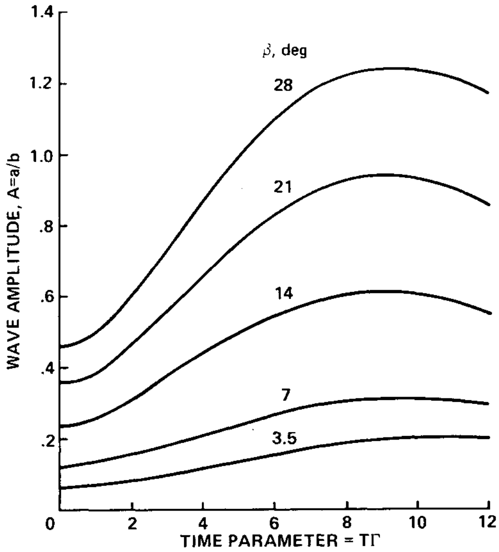
Fig. 5 Oblique and end views of a vortex pair which has been given initial sinusoidal displacement waves that are out of phase ($\phi = 180$ deg) on circular arcs as if trailed by an aircraft executing roll oscillations; wavelength = $6b$, initial wave amplitude = $0.23b$ ($\beta = 14$ deg).

the waves close together most quickly so as to benefit from the large mutual induction brought about by the proximity of the two vortices at those axial stations where the wave troughs occur. Not shown on the figures are the disconnecting and linking parts of the instability which occur after the numerical simulation has been terminated. As mentioned previously, the present method is not capable of simulating those fluid-dynamic processes.

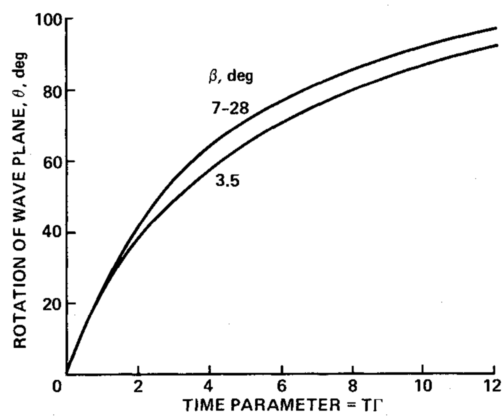
Vortex Pair: Waves Out of Phase at $\phi = 180$ deg

The disturbance waves are now initially located on circular arcs (that have a vertical secant) as if the vortices had been shed by an aircraft executing a series of roll oscillations. As a consequence, the phase of the waves on the port vortex is shifted one-half cycle $\phi = 180$ deg relative to the waves on the starboard vortex in order to study the out-of-phase or antisymmetric case. Another representation of the antisymmetric case could be made by placing the filaments in vertical planes rather than on arcs. A number of events were also calculated using such an initial configuration. It was found that the results differed only slightly and that no difference in the stability characteristics appeared. Since the circular-arc configuration more closely approximates the flight situation, only those results are presented.

The wave shape at three instances of time is presented in Fig. 5 for a typical event. The wave amplitude is noted to grow as the wave planes rotate in the retrograde direction. At the latest time shown, the wave planes have just passed the horizontal and are beginning to shrink. Also, the wave tips at the inboard ends are taking on a slight curl which contributes a small amount to the rotation angle and to the amplitude reduction. The results for a series of such events are summarized in Figs. 6a and 6b by plotting the dimensionless amplitude $A = a/b$ and the change in the angle θ of the wave planes. Figure 6a illustrates how the wave amplitude first grows to a maximum value, which occurs at about $T\Gamma = 9$, and then decreases. At about the same time (Fig. 6b), the wave planes reach the 90 deg rotation angle. Linearity of the wave dynamics for the out-of-phase wave pattern is tested in Fig. 7 by comparing the amplitude history



a) Peak-to-trough amplitude.



b) Rotation angle of wave planes from initial vertical attitude.

Fig. 6 Wave characteristics as a function of time for various input amplitudes when waves are out of phase ($\phi = 180$ deg).

for the various values of β on the basis of the input amplitude. All of the cases fall along a single line, except the small amplitude case of $\beta = 3.5$ deg. A similar observation is noted in Fig. 6b for the rotation of the wave planes.

An example of the wake vortex dynamics associated with the roll oscillation case is presented in Fig. 8. The two scenes were taken from motion pictures of the smoke trails taken during the flight tests conducted by Barber and Tymczyszyn.[5] The initial multiple-pair vortex wake of the L-1011, which is probably quite turbulent, is noted to change into a single pair of nicely formed vortices. The planes of the vortices are initially vertical, then rotate to an approximately horizontal attitude while the wave amplitude grows. Thereafter, the vortices change only a little and do so slowly. As a consequence, the two vortices then appear roughly as a parallel sinusoidal pair which could have been generated by an aircraft executing a series of yaw oscillations. These flight results confirm at least the qualitative nature of the dynamics of the vortex wake predicted by the numerical analysis.

In none of the numerical examples or flight test results is there a sign of any kind of unlimited wave growth similar to that which occurs in the symmetric or in-phase case. If wave distortions are ignored, the waves at the larger initial amplitudes appear to simply rotate, grow, and shrink in a cyclical

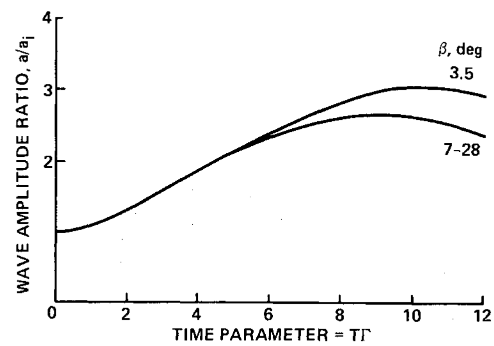
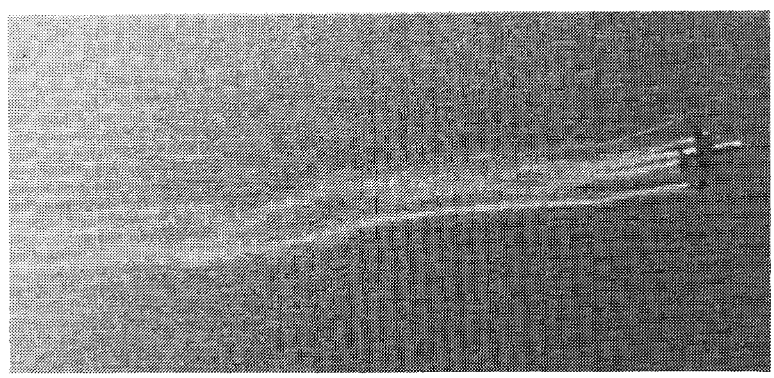


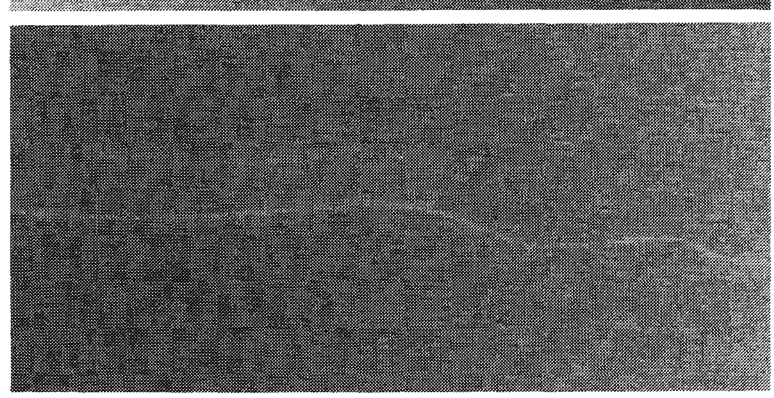
Fig. 7 Wave amplitude ratio a/a_i as a function of time to test for the linearity of the vortex dynamics to input amplitude when input waves are out of phase ($\phi = 180$ deg).

Fig. 8 Photographs of vortex wake of L-1011 following roll oscillation of about 7 deg amplitude. (From flight tests conducted by Barber and Tymczyszyn[5]; courtesy of M.R. Barber and R.M. Rhine. a) Several spanlengths behind aircraft where multiple vortex pairs initially shed by aircraft have merged into a single pair. b) About two nautical miles behind aircraft where vortex planes have rotated to horizontal attitude to produce a sinuous shape of vortices with nearly parallel axes.

a)



b)



manner without going into a destructive process. It is concluded, therefore, that the antisymmetric or out-of-phase case does not lead to any destructive vortex interactions.

Vortex Pair: Waves Initially in Horizontal Planes at $\phi = 180$ deg

The waves are now initially located in the horizontal plane as if the wake-generating aircraft had flown along a sinusoidally shaped zigzag path by executing a series of yaw oscillations. In fact, the vortex configurations presented at $T\Gamma = 6$ and 12 in Fig. 5 are approximated by the initial conditions used in this case. An example of the vortex filament shapes as a function of time is presented in Fig. 9 to illustrate how the wave amplitude decreases as the wave planes rotate away from the horizontal. The wake dynamics are then essentially the reverse of the previous or roll-oscillation case just considered (see Figs. 10a and 10b). When the wave planes have rotated past the vertical, the wave amplitude begins to grow just as was experienced in the roll-oscillation case. However, any cases with an initial amplitude near $\beta = 7$ deg experience little or no change in amplitude and little or no change in the angle of the wave plane. Once again, the wave shapes sometimes were found to undergo small distortions but devastating interactions like those of the in-phase case do not occur.

If the wave shapes were to remain nearly sinusoidal and not take on large distortions and the computer could be run long enough, the amplitude and wave plane angle would cycle continuously. Based on the information in Figs. 6b and 10b, the higher rates of rotation occur when the wave planes are vertical and slower rotation is experienced when the wave planes are horizontal. If, however, the input amplitude is around $\beta = 7$ deg, the rotation rate of the vortex planes ceases which indicates that an equilibrium condition is reached. It is coincidental that the roll amplitudes achieved with the B-747 and the L-1011 in the flight tests[5] were in this range so that the vortices took on the nearly stationary configuration[9] in some of the flight tests (see Fig. 8). As mentioned earlier, the original intent was to restrict the paper to the roll-oscillation or antisymmetric case because of the equilibrium position of the vortices at $\beta = 7$ deg that occurred in the flight tests.[5,9] Although both larger and smaller values of β bring about the continuous rotation of the wave planes, the interesting behavior near 7 deg raised the question of whether other special dynamics were being overlooked that might be of benefit in the wake-alleviation program. As it turned out, it appears that no destructive interactions are initiated by the wave patterns tried.

The equilibrium condition when $\beta \approx 7$ deg occurs because two inductive mechanisms cause the wave planes to rotate; namely, the self-induced, isolated vortex rotation rate, Fig. 1, and the rotation induced by the velocity gradient of the other vortex in the pair. When the vortex planes are approximately vertical, both mechanisms contribute positively to the rotation rate. However, when the wave planes are horizontal, the contributions to rotation are opposite one another. Hence, the rotation rate of the wave planes is greatest when the vortex planes are vertical and least when horizontal. At about $\beta = 7$ deg, the two mechanisms offset

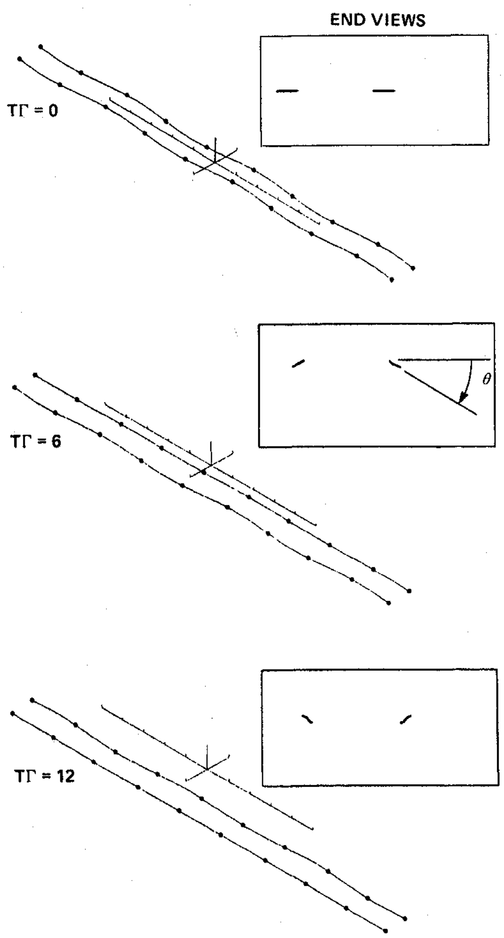
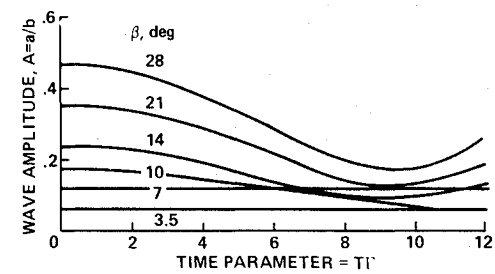
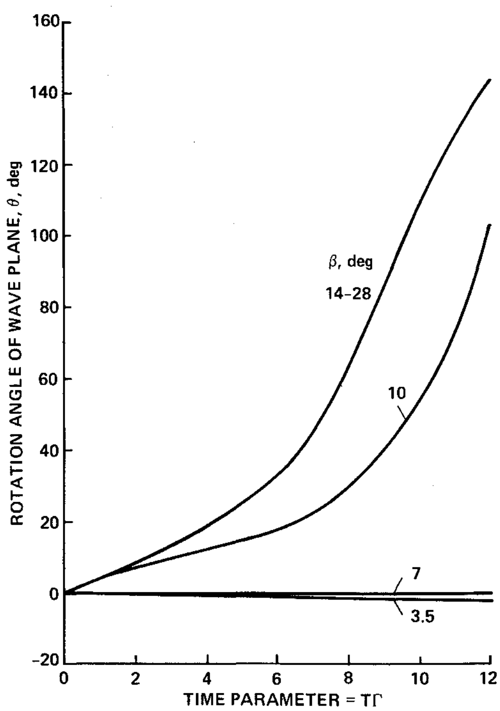


Fig. 9 Oblique and end views of a vortex pair which has been given initial sinusoidal displacements in the horizontal plane as if trailed by an aircraft executing yaw oscillations; wavelength = 6*b*, initial wave amplitude = 0.23*b*(β = 14 deg).



a) Peak-to-trough amplitude.



b) Rotation angle of wave planes from initial horizontal attitude.

Fig. 10 Wave characteristics as a function of time for various input amplitudes when input waves are out of phase and wave planes are initially in the horizontal plane as if the generating aircraft had executed a series of yaw oscillations.

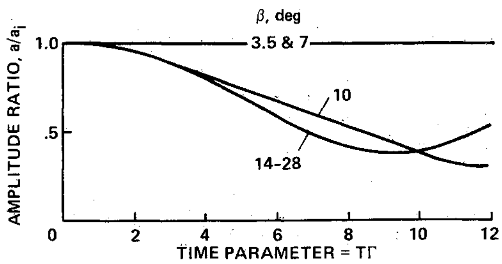


Fig. 11 Wave amplitude ratio a/a_i as a function of time to test for linearity of wake dynamics to input amplitude for the case when the waves are out of phase and initially located in a horizontal attitude.

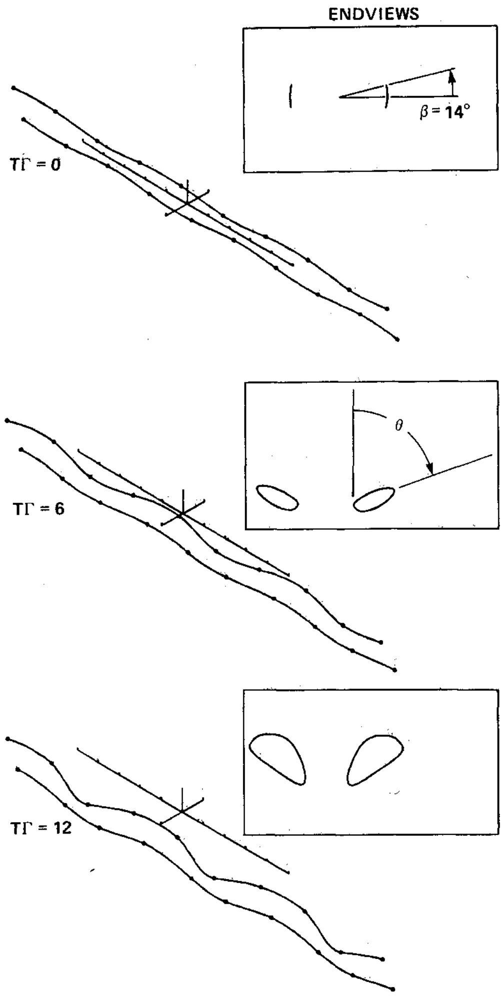


Fig. 12 Oblique and end views of vortex pair which has an initial displacement of sine waves which are midway between in phase and out of phase; $\phi = 90$ deg, wavelength $= 6b$, initial amplitude $= 0.23b(\beta = 14$ deg).

one another so that the wave plane does not rotate. When β is less than 7 deg, the wave planes rotate slowly in the opposite or posigrade direction as indicated in Fig. 10b.

As before, the linearity of the wave dynamics to input amplitude is tested by comparing the various cases with one another on the basis of the input amplitude a_i (see Fig. 11). The cases appear to fall into two groups with the transition between them occurring somewhere around $\beta = 10$ deg.

Vortex Pair: Intermediate Phase Angles 0 deg $< \phi <$ 180 deg

When the starboard vortex filament is shifted forward to yield intermediate phase angles, the vortex dynamics are a

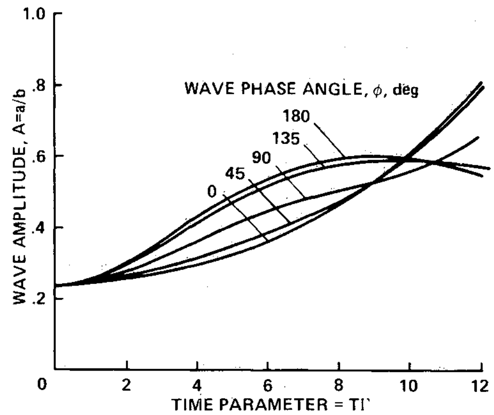


Fig. 13 Wave amplitude as a function of time for various input phase angles ϕ of the impressed wave pattern; $\beta = 14$ deg.

combination of the in-phase and out-of-phase cases. The midphase case of $\phi = 90$ deg shown in Fig. 12 illustrates the mixture of the two kinds of dynamics such that the waves appear as loops in the end views. As time progresses, the troughs or lower parts of the waves grow in amplitude in very much the same way as the in-phase case. If the numerical simulation were continued, the vortex filaments would no doubt eventually go into the disconnecting and linking parts of the instability that leads to loop formation and the disruption of the orderliness of the wakes. The curves shown for the amplitude of the input waves as a function of time (Fig. 13) illustrate how phase angle affects the growth rate for $\beta = 14$ deg. It is found that any initial input of that magnitude would eventually go into the rapid wave growth in the wave troughs which is typical of the in-phase dynamics if the phase angle is between 0 and about 150 deg. Even though the wave amplitude reaches a maximum and then begins to decrease, the vertical extent of the wave continues to grow so that the loop in the end view of the vortex opens up more and more. As time increases further, the troughs of the waves get deeper until the Scorer-Crow process of disconnecting and linking occurs. It then appears that the in-phase instability comes about for a broad range of phase angles but a longer time is required for the cases with larger phase angles just as more time is required for the smaller input wave amplitudes. During the early parts of the event, the vortex motions in general resemble more closely those of the case at one end of the range than the other depending on which is the closest.

Once again, no new destructive mechanism appears to be initiated by any of the in-between values of the phase angle. It is concluded that an out-of-phase wave value only contributes to the delay of the onset of the rapid-growth of the in-phase instability. That is, the farther that ϕ is from 0 deg, the more time is required for the onset of the rapid growth associated with the in-phase case. When the port and starboard waves are approximately out of phase, destructive wave growth may be delayed indefinitely.

Concluding Remarks

Numerical analysis of the dynamics of a vortex pair indicates that the only instability which leads to large-scale disruption of the wake organization is the process identified by Scorer.[7] The present study found that the instability occurs not only when the disturbance waves are in-phase, but also when the waves are out-of-phase by as much as 0.4 cycle or about 150 deg. However, the cases which are further out-of-phase require more time for the wave amplitude to become large. As a consequence, larger separation distances are needed for the instability to develop, go through the disconnecting and linking processes to form irregularly shaped vortex loops, and then decompose to provide the desired wake alleviation. In spite of the effectiveness of the in-phase or symmetric instability over a wide range of phase angles, its usefulness as an alleviation scheme for wakes composed of one vortex pair appears to be questionable. First, even though the vortex filaments were given substantial (and probably impractically large) initial displacements, somewhere between 50 to 100 vortex spans of travel are required before the waves begin the rapid growth sequence that leads to disconnecting and linking. Thereafter, even more time is required for the vortex loops to disorganize and disperse the wake so that it decays to a harmless level. The entire process appears to require enough distance behind large wide-body transport aircraft that the current 3–6-mile separation criteria at airports could not be substantially reduced.

If, however, the in-phase instability can be initiated in multiple vortex wakes, the distance required for wake alleviation may be in the useful range. That is, the wake interaction is still based on vortex spans; but, in the case of vortex wakes that are composed of multiple pairs, the vortex span is on the order of the flap length rather than the entire wing span. Hence, the distance required for wake alleviation through a Scorer-Crow-type vortex interaction to go to completion may be reduced by a factor of four or more. One such example was the alleviation achieved with roll oscillations of the B-747 aircraft.[9] It then remains to find those flap arrangements that result in multiple vortex instabilities and interactions which are most rapidly destructive of lift-generated wakes and which are amenable to implementation. Although partial success along these lines has been demonstrated with multiple vortex wakes,[2,5,9,11,17] systematic guidelines have not been developed for the design of vortex wakes, span loadings, or wing structures that accomplish the desired wake disruptions. Any investigation directed toward such a goal is complicated by at least the following factors: 1) the number of parameters available for variation is large; 2) the vortex interactions are a combination of several kinds of dynamics including the in-phase and out-of-phase ones; and 3) the analysis method must be capable of treating finite disturbances and large vortex distortions.

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